Since $\Gamma(n)$ has its minimum value between $n=1$ and $n=2$ it is advantageous to place $r=1$, hence

$$\Gamma(1+n) = 12n . \frac{12n-1}{1} . \frac{12n-2}{2} \ldots \frac{12n-6}{6} \left[\frac{1}{12n} - \frac{6}{12n-1} \Gamma(\tfrac{13}{12}) + \frac{15}{12n-2} \Gamma(\tfrac{7}{6}) - \frac{20}{12n-3} \Gamma(\tfrac{5}{4}) + \frac{15}{12n-4} \Gamma(\tfrac{4}{3}) - \frac{6}{12n-5} \Gamma(\tfrac{17}{12}) + \frac{1}{12n-6} \Gamma(\tfrac{3}{2}) \right], \qquad (20)$$

n being some fraction $< \frac{1}{2}$.

The following table of the gamma function was computed by the above convenient formula:

n.	$\log \Gamma(n)$.
1.000	0.0000000
1.083	9.9814951
1.166	9.9674166
1.250	9.9573211
1.333	9.9508415
1.416	9.9476700
1.500	9.9475450

and these values being used in (20) for values of $n < \frac{1}{2}$ and > 0 will give results correct to sixth differences.

NOTE ON THE METHOD OF LEAST SQUARES.

BY R. J. ADCOCK, MONMOUTH, ILL.

WHEN a greater number of points are given or observed than are sufficient to determine any point, line or surface, that point, line or surface which makes the sum of the squares of the errors of situation a minimum, has the greatest probability, and is therefore the one determined by these points.

(1). Let the coordinates, (x_1, y_1, z_1), (x_2, y_2, z_2), ... (x_n, y_n, z_n) of n points, be given or measured, and represent by d_1, d_2 ... d_n, the distances respectively from the n points to any point (α, β, γ).

If $m =$ the number of points on a unit of surface, then the probability that a point, taken at random on any of the surfaces of the spheres whose centers are (x_1, y_1, z_1), (x_2, y_2, z_2), ... (x_n, y_n, z_n), and radii respectively $d_1, d_2, \ldots d_n$, shall be at the point (α, β, γ), is $\dfrac{1}{\pi m(d_1^2+d_2^2+\ldots+d_n^2)}$, which probability is greatest when $d_1^2+d_2^2+\ldots+d_n^2 =$ a minimum.

(2). Let the coordinates (x_1, y_1, z_1), (x_2, y_2, z_2) .... (x_n, y_n, z_n), of n points be given or measured, and let δ_1, δ_2, ... δ_n, be the normals respectively from the n points to any line or surface. Then the probability that a point, taken at random on any of the surfaces of the spheres whose centers are (x_1, y_1, z_1), (x_2, y_2, z_2), ... (x_n, y_u, z_n), and radii, δ_1, δ_2, ... δ_n, shall be at the foot of one of these normals is $\dfrac{n}{\pi m(\delta_1^2+\delta_2^2+\ldots+\delta_n^2)}$. And the probability that n points, taken at random on the surfaces of the spheres, shall be at the intersection of these normals with the line or surface is

$$\frac{n^n}{\pi^n m^n(\delta_1^2+\delta_2^2+\ldots+\delta_n^2)^n},$$

which probability is greatest when $\delta_1^2+\delta_2^2+\ldots+\delta_n^2$ = a minimum.

That is, from (1) and (2), the point, line or surface, which n points make the most probable, is the point line or surface which makes the sum of the squares of the normals upon it, or sum of the squares of the errors of situation a minimum.

SOLUTIONS OF PROBLEMS IN NUMBER EIVE.

Solutions of problems in number five have been received as follows:

From Marcus Baker, 177, 178 and 179; Prof. W. P. Casey, 177 and 178; G. M. Day, 178; Prof. H. T. Eddy, 179; Edgar Frisby, 178; Newton Fitz, 175; Henry Gunder, 175, 177, 178, 179 and 180; Henry Heaton, 175, 177, 178, 179 and 180; Geo. Lilley, 179; Christine Ladd, 177; Prof. H. T. J. Ludwick, 178; Prof. D. J. Mc. Adam, 177, 178 and 179; Prof. Orson Pratt, 176; Werner Stille, 179; E. B. Seitz, 175, 177, 178, 179, 180, 181; Prof. J. Scheffer, 175, 177, 178, 179, 180; Prof. D. Trowbridge, 179.

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175. "Find the roots of the equation $x^4+Ax^3+Bx^2+Cx+C^2\div A^2=0$."

SOLUTION BY E. B. SEITZ, GREENVILLE, OHIO.

Multiplying the equation by $4A^2$, then adding $(A^4+8AC-4A^2B)x^2$ to both members, we get

$$4A^2x^4+4A^3x^3+(A^4+8AC)x^2+4A^2Cx+4C^2=(A^4+8AC-4AB)x^2.$$

Extracting the square root, we have

$$2Ax^2+A^2x+2C=\pm x\sqrt{(A^4+8AC-4A^2B)},$$ whence

$$x=\{-A^2\pm\sqrt{(A^4+8AC-4A^2B)}\pm A\sqrt{[2A^2-4B\mp2\sqrt{(A^4+8AC-4A^2B)}]}\}\div4A.$$